Tutorial Worksheet 7

1) Check the proof of Theorem 4.5.4 again. The theorem states: every natural number n can be expressed as a sum of distinct nonnegative integer powers of 2. Explain why it is essential to apply strong induction.

2) Check the proof for Theorem 4.5.1 again. The theorem states: every natural number $n \ge 2$ can be written as a product of prime numbers. Explain how the induction hypothesis is applied to prove the claim.

3) Let (a_n) be a sequence satisfying $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \ge 3$. Given that a_1 and a_2 are odd, prove that a_n is odd for $n \in \mathbb{N}$.